Christina School District Assignment Board

Grade Level: 12th

Week of April 20th, 2020

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<tr><th></th><th>Day 1</th><th>Day 2</th><th>Day 3</th><th>Day 4</th><th>Day 5</th></tr>
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<td>ELA</td>
<td>In your own words create a definition for the words vanity and self-confidence. Next, take a sheet of paper and make two columns. In each column write down 3-5 examples of vanity and 3-5 examples of self-confidence. Review your list.
What is the difference between vanity and self-confidence?</td>
<td>Background: Aristocracy means the highest class of society, the wealthy or upper class.
Read “The Rape of the Lock” by Alexander Pope. As you read, underline all instances of Pope mocking or poking fun at the aristocracy.</td>
<td>Answer the following on a separate sheet of paper.
1. What happens in the card game in lines 29–54?
2. How does the Baron obtain the lock of Belinda’s hair?
3. At the end of the poem, what happens to the lock of Belinda’s hair?
4. A contrast between expectations and actual outcomes is called situational irony. Where is the irony in The Rape of the Lock?</td>
<td>The Rape of the Lock parodies the epic form by treating a trivial subject in a grand, lofty style. Create a chart similar to the one below.
In each column, citing specific examples from the text, describe how Pope makes fun of these elements of traditional epic poetry:
• elaborate descriptions of weapons and battles
• plot affected by supernatural intervention
• boasting speeches
<table>
<tr><th>Elements of Traditional Epic Poetry</th><th>Explanation & evidence</th></tr>
<tr><td>elaborate descriptions of weapons and battles</td><td></td></tr>
<tr><td>plot affected by supernatural intervention</td><td></td></tr>
<tr><td>boasting speeches</td><td></td></tr>
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</td>
<td>Pope’s friend Jonathan Swift once wrote, “Satire is a sort of glass, wherein beholders do generally discover everybody’s face but their own.” While the mock epic The Rape of the Lock was written nearly 300 years ago to poke fun at vanity, beauty, and pride, in what ways does the satire reflect today’s society?</td>
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				5. In addition to satirizing a quarrel, Pope used The Rape of the Lock to point out flaws in British society and upper-class behavior. For each of the following passages, describe the flaw that Pope is criticizing: • lines 15–16 (“A third interprets . . . dies.”) • lines 21–22 (“The hungry judges . . . dine;”) • lines 111–114 (“Not louder shrieks . . . lie!”)		
Math	**IM4**	*Algebraic & Geometric Sequences* Read pages 151-155. (attached) Use the examples as a guide. Complete page 156 #1-14. (attached)	Use the examples from pages 151-155 as a guide to complete p. 156 #15-28. (attached)	Use the examples from pages 151-155 as a guide to complete p. 157 #29-38. (attached)	Use the examples from pages 151-155 as a guide to complete p. 157 #39-50. (attached)	Use the examples from pages 151-155 as a guide to complete p. 157 #51-56. (attached)
	PreCalc	Trigonometric Functions of Any Angle Review 4.4. PP and examples to complete Reference Angle Worksheet #1.	Use 4.4 PP notes and examples to complete Reference Angle WS #2. (attached)	Use 4.4 PP notes and Unit Circle notes to complete Trig Functions Chart. (attached)	Use 4.4 PP, Unit Circle Notes, and Trig Functions Chart to complete Unit Circle Worksheet #1-5. (attached)	Use 4.4 PP, Unit Circle Notes, and Trig Functions Chart to complete Unit Circle Worksheet #6-10. (attached)

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		(attached)				
	Calc	*Linearization and Differentials*				
Science		**Defining Kinematics:** Read passage. Highlight, underline, and/or notate for understanding.	**Reference Frames and Displacement:** Read passage. Highlight, underline, and/or notate for understanding.	**Acceleration:** Read passage. Highlight, underline, and/or notate for understanding.	**Calculating the Average Acceleration:** Read passage. Highlight, underline, and/or notate for understanding.	**Direction of Acceleration Vector:** Read passage. Highlight, underline, and/or notate for understanding. Complete "Check Your Understanding" at end of passage.
Social Studies		Complete Activity 4 from the document titled, "Maine Explosion." You have this document from last week.	Complete Activity 1 & 2 from the document titled, "Spanish American War Inquiry."	Complete Activity 3 from the document titled, "Spanish American War Inquiry."	Complete Activity 4 from the document titled, "Spanish American War Inquiry."	Complete Activity 5 from the document titled, "Spanish American War Inquiry." NOTE: Activity 6 will be on next week's CSD Assignment Board.

THE RAPE OF THE LOCK

Alexander Pope

BACKGROUND *The Rape of the Lock* was based on a real-life quarrel between two affluent Roman Catholic families, the Fermors and the Petres. The feud began when young Lord Petre (the "Baron" in the poem) snipped a lock of hair from Arabella Fermor ("Belinda"). The dispute escalated out of all proportion, and a friend of Pope's asked him to intervene, hoping that he could "laugh them together again." Pope rose to the occasion, mocking the folly of the dispute by portraying it as if it were a battle of epic scale.

In the first of the poem's five cantos, a Muse is evoked for inspiration (a tradition in epic poetry) and Belinda is warned of impending danger by Ariel, a spirit sent to protect Belinda. In Canto 2, Belinda rides up the Thames River to a Hampton Court party and is noticed by the scheming Baron, who resolves to possess one of the two curly locks spiraling down Belinda's back.

from CANTO 3

Close by those meads, forever crowned with flowers,
Where Thames with pride surveys his rising towers,
There stands a structure of majestic frame,
Which from the neighboring Hampton takes its name.
Here Britain's statesmen oft the fall foredoom
Of foreign tyrants and of nymphs at home;
Here thou, great Anna! whom three realms obey,
Dost sometimes counsel take—and sometimes tea. **A**
Hither the heroes and the nymphs resort,
To taste awhile the pleasures of a court;
In various talk the instructive hours they passed,
Who gave the ball, or paid the visit last;
One speaks the glory of the British Queen,
And one describes a charming Indian screen;

1 meads: meadows.

2 Thames (tĕmz): a river that flows through southern England.

3–4 structure . . . name: the royal palace of Hampton Court, about 15 miles from London.

6 nymphs (nĭmfs): maidens; young women.

7 Anna . . . obey: Queen Anne, who rules over the three realms of England, Scotland, and Wales.

A HEROIC COUPLET
In Pope's time, *tea* was pronounced "tay." How does Pope use rhyme in lines 7–8 to mock pomposity?

The Toilet (1896), Aubrey Beardsley. Drawing for Alexander Pope's *The Rape of the Lock.* The Granger Collection.

A third interprets motions, looks, and eyes;
At every word a reputation dies.
Snuff, or the fan, supply each pause of chat,
With singing, laughing, ogling, and all that.
Meanwhile declining from the noon of day,
The sun obliquely shoots his burning ray;
The hungry judges soon the sentence sign,
And wretches hang that jurymen may dine;
The merchant from the Exchange returns in peace,
And the long labors of the toilet cease.
Belinda now, whom thirst of fame invites,
Burns to encounter two adventurous knights,
At ombre singly to decide their doom,
And swells her breast with conquests yet to come. . . .
The Baron now his Diamonds pours apace;
The embroidered King who shows but half his face,
And his refulgent Queen, with powers combined,
Of broken troops an easy conquest find.
Clubs, Diamonds, Hearts, in wild disorder seen,
With throngs promiscuous strew the level green.
Thus when dispersed a routed army runs,
Of Asia's troops, and Afric's sable sons,
With like confusion different nations fly,
Of various habit, and of various dye,
The pierced battalions disunited fall
In heaps on heaps; one fate o'erwhelms them all.
The Knave of Diamonds tries his wily arts,
And wins (oh, shameful chance!) the Queen of Hearts.
At this, the blood the virgin's cheek forsook,
A livid paleness spreads o'er all her look;
She sees, and trembles at the approaching ill,
Just in the jaws of ruin, and Codille.
And now (as oft in some distempered state)
On one nice trick depends the general fate.
An Ace of Hearts steps forth: The King unseen
Lurked in her hand, and mourned his captive Queen.
He springs to vengeance with an eager pace,
And falls like thunder on the prostrate Ace.
The nymph exulting fills with shouts the sky,
The walls, the woods, and long canals reply. **B**
O thoughtless mortals! ever blind to fate,
Too soon dejected, and too soon elate:
Sudden these honors shall be snatched away,
And cursed forever this victorious day.
For lo! the board with cups and spoons is crowned,
The berries crackle, and the mill turns round;

17 snuff: powdered tobacco that is inhaled.

24 toilet: the process of dressing, fixing one's hair, and otherwise grooming oneself.

27 ombre (ŏm′bər): a popular card game of the day, similar to bridge.

30 King . . . face: the king of diamonds, the only king shown in profile in a deck of cards.

31 refulgent (rĭ-fo͝ol′jənt) **Queen:** resplendent or shining queen of diamonds. The Baron is leading his highest diamonds in an effort to win.

34 promiscuous (prə-mĭs′kyo͞o-əs): unsorted; **level green:** the green cloth-covered card table.

36 Afric's sable sons: Africa's black soldiers.

41 Knave: jack.

43 the virgin's: Belinda's.

46 Codille (kō-dēl′): a losing hand of cards in ombre.

47 distempered: disordered.

48 nice: delicate; subtle; **trick:** a single round of cards played and won.

B ELEVATED LANGUAGE
Reread lines 53–54, imagining the sounds that Pope describes. Write a **paraphrase** of this couplet.

60 berries: coffee beans.

On shining altars of Japan they raise
The silver lamp; the fiery spirits blaze:
From silver spouts the grateful liquors glide,
While China's earth receives the smoking tide.
At once they gratify their scent and taste,
And frequent cups prolong the rich repast.
Straight hover round the fair her airy band;
Some, as she sipped, the fuming liquor fanned,
Some o'er her lap their careful plumes displayed,
Trembling, and conscious of the rich brocade.
Coffee (which makes the politician wise,
And see through all things with his half-shut eyes)
Sent up in vapors to the Baron's brain
New stratagems, the radiant Lock to gain.
Ah, cease, rash youth! desist ere 'tis too late,
Fear the just Gods, and think of Scylla's fate!
Changed to a bird, and sent to flit in air,
She dearly pays for Nisus' injured hair!

But when to mischief mortals bend their will,
How soon they find fit instruments of ill!
Just then, Clarissa drew with tempting grace
A two-edged weapon from her shining case:
So ladies in romance assist their knight,
Present the spear, and arm him for the fight.
He takes the gift with reverence, and extends
The little engine on his fingers' ends;
This just behind Belinda's neck he spread,
As o'er the fragrant steams she bends her head.
Swift to the Lock a thousand sprights repair,
A thousand wings, by turns, blow back the hair,
And thrice they twitched the diamond in her ear,
Thrice she looked back, and thrice the foe drew near.
Just in that instant, anxious Ariel sought
The close recesses of the virgin's thought;
As on the nosegay in her breast reclined,
He watched the ideas rising in her mind,
Sudden he viewed, in spite of all her art,
An earthly lover lurking at her heart.
Amazed, confused, he found his power expired,
Resigned to fate, and with a sigh retired.

The Peer now spreads the glittering forfex wide,
To enclose the Lock; now joins it, to divide.
Even then, before the fatal engine closed,
A wretched Sylph too fondly interposed;
Fate urged the shears, and cut the Sylph in twain

61 shining altars of Japan: small lacquered tables. In mock-epic style, Pope elevates the tables to altars.

64 China's earth . . . tide: China cups receive the hot coffee.

66 repast (rĭ-păst′): meal.

67 the fair: Belinda; **her airy band:** the Sylphs (sĭlfs), supernatural creatures attending Belinda. Epic heroes and heroines are generally aided by higher powers.

74 new stratagems (străt′ə-jəmz) **. . . gain:** new schemes for acquiring a lock of Belinda's hair.

76–78 Scylla's (sĭl′əz) **fate . . . Nisus'** (nī′səs) **injured hair:** In ancient Greek legend, Scylla was turned into a bird because she betrayed her father, King Nisus, by giving his enemy the purple lock of his hair on which his safety depended.

Language Coach

Figurative Language In lines 81–86, Pope refers to an everyday object through metaphors: *weapon, spear,* and *engine*. What do these metaphors refer to?

89 sprights (sprīts): the Sylphs.

93 Ariel (âr′ē-əl): Belinda's special guardian among the Sylphs.

95 nosegay: a small bouquet of flowers.

101 the Peer: the Baron; **forfex:** a fancy term for scissors.

The Rape (1896), Aubrey Beardsley. From *The Rape of the Lock* by Alexander Pope. Line block print. CT46089. Victoria & Albert Museum, London.

(But airy substance soon unites again):
The meeting points the sacred hair dissever
From the fair head, forever and forever!
     Then flashed the living lightning from her eyes,
And screams of horror rend the affrighted skies. C
Not louder shrieks to pitying heaven are cast,
When husbands, or when lapdogs breathe their last;
Or when rich china vessels fallen from high,
In glittering dust and painted fragments lie!
"Let wreaths of triumph now my temples twine,"
The victor cried, "the glorious prize is mine!
While fish in streams, or birds delight in air,
Or in a coach and six the British fair,
As long as *Atalantis* shall be read,
Or the small pillow grace a lady's bed,
While visits shall be paid on solemn days,
When numerous wax-lights in bright order blaze,

C HEROIC COUPLET
Reread lines 107–110. Which details in these couplets highlight the contrast between the actual incident that occurs and Belinda's exaggerated reaction?

115 wreaths . . . twine: In epics, victors or champions traditionally wore laurel wreaths as a kind of crown.

118 coach and six: a coach drawn by six horses.

119 *Atalantis:* *The New Atalantis* by Mary Manley, a thinly disguised account of scandal among the rich.

While nymphs take treats, or assignations give,
So long my honor, name, and praise shall live!
"What time would spare, from steel receives its date,
And monuments, like men, submit to fate!
Steel could the labor of the Gods destroy,
And strike to dust the imperial towers of Troy;
Steel could the works of mortal pride confound,
And hew triumphal arches to the ground.
What wonder then, fair nymph! thy hairs should feel,
The conquering force of unresisted steel?" **D**

In Canto 4, following an epic tradition, a melancholy sprite descends to the Underworld—which Pope calls the "Cave of Spleen"—and returns to the party with a vial of grief and "flowing tears" and a bag of "sobs, sighs, and passions," which are emptied over Belinda's head, fanning her fury even further.

from CANTO 5

"To arms, to arms!" the fierce virago cries,
And swift as lightning to the combat flies.
All side in parties, and begin the attack;
Fans clap, silks rustle, and tough whalebones crack;
Heroes' and heroines' shouts confusedly rise,
And bass and treble voices strike the skies.
No common weapons in their hands are found,
Like Gods they fight, nor dread a mortal wound. . . . **E**
See, fierce Belinda on the Baron flies,
With more than usual lightning in her eyes;
Nor feared the chief the unequal fight to try,
Who sought no more than on his foe to die.
But this bold lord with manly strength endued,
She with one finger and a thumb subdued:
Just where the breath of life his nostrils drew,
A charge of snuff the wily virgin threw;
The Gnomes direct, to every atom just,
The pungent grains of titillating dust.
Sudden, with starting tears each eye o'erflows,
And the high dome re-echoes to his nose.
"Now meet thy fate," incensed Belinda cried,
And drew a deadly bodkin from her side.
(The same, his ancient personage to deck,
Her great-great-grandsire wore about his neck,
In three seal rings; which after, melted down,
Formed a vast buckle for his widow's gown:

125 date: end.

127–128 the labor of the Gods . . . towers of Troy: Troy, an ancient city famous for its towers, whose walls were said to have been built by the Greek gods Apollo and Poseidon.

D MOCK EPIC
In lines 125–132, what humorous effect does Pope create by using lofty language and allusions to Greek mythology?

133 virago (və-räʹgō): a woman who engages in warfare or other fighting. She has come to Belinda's aid at Ariel's request.

136 whalebones: elastic material from whales' mouths, used in corsets or support undergarments.

E MOCK EPIC
What characteristics of a mock epic do you find in lines 133–140?

145 endued (ĕn-do͞odʹ): endowed; provided with.

149 Gnomes (nōmz): supernatural creatures bent on causing mischief.

152 And the high . . . nose: In other words, he sneezes.

154 bodkin (bŏdʹkĭn): a long, ornamental hairpin.

157 seal rings: signet rings bearing a person's family crest or initials.

Her infant grandame's whistle next it grew,
The bells she jingled, and the whistle blew;
Then in a bodkin graced her mother's hairs,
Which long she wore, and now Belinda wears.)
"Boast not my fall," he cried, "insulting foe!
Thou by some other shalt be laid as low.
Nor think to die dejects my lofty mind:
All that I dread is leaving you behind!
Rather than so, ah, let me still survive,
And burn in Cupid's flames—but burn alive."
"Restore the Lock!" she cries; and all around
"Restore the Lock!" the vaulted roofs rebound.
Not fierce Othello in so loud a strain
Roared for the handkerchief that caused his pain.
But see how oft ambitious aims are crossed,
And chiefs contend till all the prize is lost!
The lock, obtained with guilt, and kept with pain,
In every place is sought, but sought in vain:
With such a prize no mortal must be blessed,
So Heaven decrees! with Heaven who can contest?
Some thought it mounted to the lunar sphere,
Since all things lost on earth are treasured there.
There heroes' wits are kept in ponderous vases,
And beaux' in snuffboxes and tweezer cases.
There broken vows and death-bed alms are found,
And lovers' hearts with ends of riband bound. . . .
But trust the Muse—she saw it upward rise,
Though marked by none but quick, poetic eyes. . . .
A sudden star, it shot through liquid air,
And drew behind a radiant trail of hair. . . .
Then cease, bright nymph! to mourn thy ravished hair,
Which adds new glory to the shining sphere!
Not all the tresses that fair head can boast
Shall draw such envy as the Lock you lost.
For, after all the murders of your eye,
When, after millions slain, yourself shall die:
When those fair suns shall set, as set they must,
And all those tresses shall be laid in dust,
This Lock the Muse shall consecrate to fame,
And 'midst the stars inscribe Belinda's name. F

159 Her infant grandame's (grăn′dāmz) **. . . grew:** It was next melted down and turned into a whistle used by Belinda's grandmother as a child. Pope is here making fun of family heirlooms.

168 burn in Cupid's flames: burn with passion.

170 rebound: echo.

171–172 Othello . . . pain: In Shakespeare's *Othello,* the deeply jealous Othello demands the handkerchief that he believes is a sign of his wife's infidelity.

179 mounted to the lunar sphere: climbed up to the moon.

182 beaux' (bōz): the wits of fops.

184 riband (rĭb′ənd): ribbon.

185 Muse (myo͞oz): the goddess who inspires the writing of the poem. In typical epic fashion, the narrator opens the poem by addressing his Muse and continues to address her throughout the poem.

188 trail of hair: The word *comet* comes from a Greek word that means "long haired."

193 murders of your eye: men struck down by your glance.

F ELEVATED LANGUAGE
Reread lines 193–198 and the accompanying side note. **Paraphrase** what the narrator says to comfort Belinda about the loss of her lock.

IM4 – Week of April 20th

Algebraic & Geometric Sequences

EQUATIONS FOR SEQUENCES — A.2.1 – A.2.3

In these lessons, students learn multiple representations for sequences: as a string of numbers, as a table, as a graph, and as an equation. Read more about writing equations for sequences in the Math Notes box in Lesson B.2.3.

In addition to the ways to write explicit equations for sequences, as explained in the Math Notes box in Lesson B.2.3, equations for sequences less-commonly are written recursively. An explicit formula tells exactly how to find any specific term in the sequence. A recursive formula names the first term (or any other term) and how to get from one term to the next. For an explanation of recursive sequences, see the Math Notes box in Lesson A.3.2. For additional examples and more practice, see the Checkpoint 4A materials in the student textbook.

Example 1

This is the same scenario as in Example 1 of the previous section, *Introduction to Sequences*.

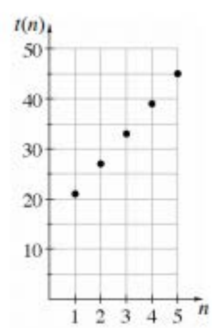
Peachy Orchard Developers are preparing land to create a large subdivision of single-family homes. They have already built 15 houses on the site. Peachy Orchard plans to build six new homes every month. Write a sequence for the number of houses built, then write an equation for the sequence. Fully describe a graph of this sequence.

The sequence is $21, 27, 33, 39, \ldots$. Note that sequences usually begin with the first term, where the number of months $n = 1$.

The common difference is $m = 6$, and the zeroth term is $b = 15$. The equation can be written $t(n) = mn + b = 6n + 15$. Note that for a sequence, $t(n) =$ is used instead of $y =$. $t(n) =$ indicates the equation is for a discrete sequence, as opposed to a continuous function. Students compared sequences to functions in Lesson 5.3.3.

The equation could also have been written as $a_n = 6n + 15$.

The graph of the sequence is shown at right. There are no x- or y-intercepts. There is no point at $(0, 15)$ because sequences are usually written starting with the *first* term where $n = 1$ The domain consists of *integers* (whole numbers) greater than or equal to one. The range consists of the y-values of the points that follow the rule $t(n) = 6n + 15$ when $n \geq 1$. There are no asymptotes. The graph is linear and is shown at right. *This graph is discrete* (separate points). Note: The related function, $y = 6x + 15$, would have the domain of all real numbers (including fractions and negatives) and the graph would be a continuous connected line.



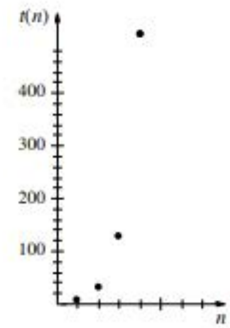
Example 2

This is the same scenario as in Example 2 of the previous section, *Introduction to Sequences*.

When Rosa tripped and fell into a muddy puddle at lunch (she was so embarrassed!), she knew exactly what would happen: within ten minutes, the two girls who saw her fall would each tell four people what they had seen. Within the next ten minutes, those eight students would each tell four more people. Rosa knew this would continue until everyone in the entire school was talking about her muddy experience. Write a sequence for the number of people who knew about Rosa mishap in ten-minute intervals, then write an equation for the sequence. Fully describe a graph of this sequence.

The multiplier is $b = 4$, and the zeroth term is $a = 2$. The equation can be written $t(n) = ab^n = 2 \cdot 4^n$. The equation could also have been written as $a_n = 2 \cdot 4^n$. (Later, in Appendix B, students will also learn "first term" notation for sequences, $a_n = 8 \cdot 4^{(n-1)}$.)

The sequence is: $8, 32, 128, 512, \ldots$. Note that the sequence is written starting with $n = 1$.



The graph of the sequence is to the right. There are no x- or y-intercepts. There is no point at $(0, 2)$ because sequences are usually written starting with the *first* term where $n = 1$. The domain consists of *integers* (whole numbers) greater than or equal to one. The range consists of the y-values of the points that follow the rule $t(n) = 2(4)^n$ when $n \geq 1$. The graph is exponential and is shown at right. There is no symmetry. This graph is discrete (separate points). (Note: The related function, $y = 2 \cdot 4^n$ would have the domain of all real numbers (including fractions and negatives) and the graph would be a solid curve.)

Example 3

Consider the two sequences:

A: $-8,\ -5,\ -2,\ 1, \ldots$
B: $256,\ 128,\ 64, \ldots$

a. For each sequence, is it arithmetic, geometric, or neither? How can you tell? Explain completely.

b. What are the zeroth term and the generator for each sequence?

c. For each sequence, write an equation representing the sequence.

d. Is 378 a term of sequence A? Justify your answer.

e. Is $\frac{1}{4}$ a term of sequence B? Justify your answer.

To determine the type of sequence for A and B above, we have to look at the growth of each sequence.

```
A:  -8,  -5,  -2,  1,   ...
      \  / \  / \  /
       +3   +3   +3
```

Sequence A is made (generated) by adding three to each term to get the next term. When each term has a **common difference** (in this case, "+3") the sequence is **arithmetic**.

Sequence B, however, is different. The terms do not have a common difference.

```
B:  256, 128, 64,  ...
       \  / \  /
       -128  -64
```

Instead, these terms have a **common ratio** (multiplier). A sequence with a common ratio is a **geometric sequence**.

B: $256,\ 128,\ 64, \ldots$
$\cdot \frac{1}{2} \quad \cdot \frac{1}{2}$

The first term for sequence A is -8, and has a generator or common difference of $+3$. Therefore the zeroth term is -11 (because $-11 + 3 = -8$). An arithmetic sequence has an equation of the form $t(n) = mn + b$ (or $a_n = mn + a_0$) where m is the common difference, and b is the initial value. For sequence A, the equation is $t(n) = 3n - 11$, for $n = 1, 2, 3, \ldots$

Example continues on next page →

Example continued from previous page.

For sequence B, the first term is 256 with a generator or common ratio of $\frac{1}{2}$. Therefore the zeroth term is 512, because $512 \cdot \frac{1}{2} = 256$. The general equation for a geometric sequence is $t(n) = ab^n$ where a is the zeroth term, and b is the common ratio (multiplier). For sequence B, the equation is $t(n) = 512\left(\frac{1}{2}\right)^n$ for $n = 1, 2, 3, \ldots$

To check if 378 is a term in sequence A, we could list the terms of the sequence out far enough to check, but that would be time consuming. Instead, we will check if there is an integer n that solves $t(n) = 3n - 11 = 378$.

$$\begin{aligned} 3n - 11 &= 378 \\ 3n &= 389 \\ n &= \tfrac{389}{3} = 129\tfrac{2}{3} \end{aligned}$$

When we solve, n is not a whole number, therefore 378 cannot be a term in the sequence.

Similarly, to check if $\frac{1}{4}$ is a term in sequence B, we need to solve $t(n) = 512(\frac{1}{2})^n = \frac{1}{4}$, and look for a whole number solution.

$$\begin{aligned} 512\left(\tfrac{1}{2}\right)^n &= \tfrac{1}{4} \\ \tfrac{1}{512} \cdot 512\left(\tfrac{1}{2}\right)^n &= \tfrac{1}{512} \cdot \tfrac{1}{4} \\ \left(\tfrac{1}{2}\right)^n &= \tfrac{1}{2048} \\ \left(\tfrac{1}{2}\right)^n &= \tfrac{1}{2^{11}} \\ \tfrac{1}{2^n} &= \tfrac{1}{2^{11}} \\ n &= 11 \end{aligned}$$

Although solving an equation like this is probably new for most students, they can solve this problem by using guess-and-check. Also, by writing both sides as a power of 2, students can see the solution easily.

Since the equation has a whole number solution, $\frac{1}{4}$ is a term of sequence B.

That is, when $n = 11$, $t(n) = \frac{1}{4}$.

Examples For Arithmetic Sequences

List the first five terms of each arithmetic sequence.

Example 4 (An explicit formula)

$$t(n) = 5n + 2$$

$t(1) = 5(1) + 2 = 7$
$t(2) = 5(2) + 2 = 12$
$t(3) = 5(3) + 2 = 17$
$t(4) = 5(4) + 2 = 22$
$t(5) = 5(5) + 2 = 27$

The sequence is: $7, 12, 17, 22, 27, \ldots$

Example 5 (A recursive formula)

$$t(1) = 3,\ t(n+1) = t(n) - 5$$

$t(1) = 3$
$t(2) = t(1) - 5 = 3 - 5 = -2$
$t(3) = t(2) - 5 = -2 - 5 = -7$
$t(4) = t(3) - 5 = -7 - 5 = -12$
$t(5) = t(4) - 5 = -12 - 5 = -17$

The sequence is: $3, -2, -7, -12, -17, \ldots$

Example 6

Find an explicit and a recursive formula for the sequence: $-2, 1, 4, 7, \ldots$

Explicit: $m = 3$, $b = -5$ so the equation is: $t(n) = mn + b = 3n - 5$

Recursive: $t(1) = -2$, $t(n + 1) = t(n) + 3$

Examples For Geometric Sequences

List the first five terms of each geometric sequence.

Example 7 (An explicit formula)

$$t(n) = 3 \cdot 2^{n-1}$$

$t(1) = 3 \cdot 2^{1-1} = 3 \cdot 2^{0} = 3$

$t(2) = 3 \cdot 2^{2-1} = 3 \cdot 2^{1} = 6$

$t(3) = 3 \cdot 2^{3-1} = 3 \cdot 2^{2} = 12$

$t(4) = 3 \cdot 2^{4-1} = 3 \cdot 2^{3} = 24$

$t(5) = 3 \cdot 2^{5-1} = 3 \cdot 2^{4} = 48$

The sequence is: $3, 6, 12, 24, 48, \ldots$

Example 8 (A recursive formula)

$$t(1) = 8,\ t(n+1) = t(n) \cdot \tfrac{1}{2}$$

$t(1) = 8$

$t(2) = t(1) \cdot \frac{1}{2} = 8 \cdot \frac{1}{2} = 4$

$t(3) = t(2) \cdot \frac{1}{2} = 4 \cdot \frac{1}{2} = 2$

$t(4) = t(3) \cdot \frac{1}{2} = 2 \cdot \frac{1}{2} = 1$

$t(5) = t(4) \cdot \frac{1}{2} = 1 \cdot \frac{1}{2} = \frac{1}{2}$

The sequence is: $8, 4, 2, 1, \frac{1}{2}, \ldots$

Example 9

Find an explicit and a recursive formula for the sequence: $81, 27, 9, 3, \ldots$

Explicit: $a_1 = 81$, $b = \frac{1}{3}$ so the a_0 (the zeroth term) is found by $a_0 = 81 \div \frac{1}{3} = 243$ and the answer is: $a_n = a_0 \cdot b^n = 243 \cdot \left(\frac{1}{3}\right)^n$, or alternatively, $t(n) = 243 \cdot \frac{1}{3}^n$

Recursive: $t(1) = 81,\ t(n+1) = t(n) \cdot \frac{1}{3}$

Problems

Each of the functions listed below defines a sequence. List the first five terms of the sequence, and state whether the sequence is arithmetic, geometric, both, or neither.

1. $t(n) = 5n + 2$
2. $s_n = 3 - 8n$
3. $u(n) = 9n - n^2$
4. $t(n) = (-4)^n$
5. $s(n) = \left(\frac{1}{4}\right)^n$
6. $u(n) = n(n+1)$
7. $t(n) = 8$
8. $s_n = \frac{3}{4}n + 1$

Identify each of the following sequences as arithmetic or geometric. Then write the equation that gives the terms of the sequence.

9. $48, 24, 12, 6, 3, \ldots$
10. $-4, 3, 10, 17, 24, \ldots$
11. $43, 39, 35, 31, 27, \ldots$
12. $\frac{2}{3}, \frac{1}{2}, \frac{3}{8}, \frac{9}{32}, \frac{27}{128}, \ldots$
13. $5, -5, 5, -5, 5, \ldots$
14. $10, 1, 0.1, 0.01, 0.001, \ldots$

Graph the following two sequences on the same set of axes.

15. $t(n) = -6n + 20$
16. $1, 4, 16, 64, \ldots$
17. Do the two sequences of the last two problems have any terms in common? Explain how you know.
18. Every year since 1548, the average height of a human male has increased slightly. The new height is 100.05% of what it was the previous year. If the average male's height was 54 inches in 1548, what is the average height of a male in 2008?
19. Davis has \$5.40 in his bank account on his fourth birthday. If his parents add \$0.40 to his bank account every week, when will he have enough to buy the new Smokin' Derby race car set which retails for \$24.99?
20. Fully describe the graph of the sequence $t(n) = -4n + 18$.

Arithmetic Sequences

List the first five terms of each arithmetic sequence.

21. $t(n) = 5n - 2$
22. $t(n) = -3n + 5$
23. $t(n) = -15 + \frac{1}{2}n$
24. $t(n) = 5 + 3(n - 1)$
25. $t(1) = 5,\ t(n+1) = t(n+3)$
26. $t(1) = 5,\ t(n+1) = t(n) - 3$
27. $t(1) = -3,\ t(n+1) = t(n) + 6$
28. $t(1) = \frac{1}{3},\ t(a+1) = t(n) + \frac{1}{2}$

Find the 30$^{\text{th}}$ term of each arithmetic sequence.

29. $t(n) = 5n - 2$

30. $t(n) = -15 + \frac{1}{2}n$

31. $t(31) = 53,\ d = 5$

32. $t(1) = 25,\ t(n+1) = t(n) - 3$

For each arithmetic sequence, find an explicit and a recursive formula.

33. $4, 8, 12, 16, 20, \ldots$

34. $-2, 5, 12, 19, 26, \ldots$

35. $27, 15, 3, -9, -21, \ldots$

36. $3, 3\frac{1}{3}, 3\frac{2}{3}, 4, 4\frac{1}{3}, \ldots$

Sequences are graphed using points of the form: (term number, term value).
For example, the sequence 4, 9, 16, 25, 36, … would be graphed by plotting the points $(1, 4)$, $(2, 9), (3, 16), (4, 25), (5, 36), \ldots$. Sequences are graphed as points and not connected.

37. Graph the sequences from problems 1 and 2 above. What are the slopes of the lines determined by the points?

38. How do the slopes of the lines found in the previous problem relate to the sequences?

Geometric Sequences

List the first five terms of each geometric sequence.

39. $t(n) = 5 \cdot 2^n$

40. $t(n) = -3 \cdot 3^n$

41. $t(n) = 40\left(\frac{1}{2}\right)^{n-1}$

42. $t(n) = 6\left(-\frac{1}{2}\right)^{n-1}$

43. $t(1) = 5,\ t(n+1) = t(n) \cdot 3$

44. $t(1) = 100,\ t(n+1) = t(n) \cdot \frac{1}{2}$

45. $t(1) = -3,\ t(n+1) = t(n) \cdot (-2)$

46. $t(1) = \frac{1}{3},\ t(n+1) = t(n) \cdot \frac{1}{2}$

Find the 15$^{\text{th}}$ term of each geometric sequence.

47. $t(14) = 232,\ r = 2$

48. $t(16) = 32,\ r = 2$

49. $t(14) = 9,\ r = \frac{2}{3}$

50. $t(16) = 9,\ r = \frac{2}{3}$

Find an explicit and a recursive formula for each geometric sequence.

51. $2, 10, 50, 250, 1250, \ldots$

52. $16, 4, 1, \frac{1}{4}, \frac{1}{16}, \ldots$

53. $5, 15, 45, 135, 405, \ldots$

54. $3, -6, 12, -24, 48, \ldots$

55. Graph the sequences from problems 39 and 52. Remember the note before problem 37 about graphing sequences.

56. How are the graphs of geometric sequences different from the graphs of arithmetic sequences?

PreCalculus - 4.4 Introduction

Following is the definition of trigonometric functions of Any Angle.

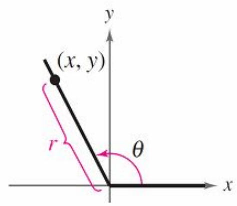
Definition of Trigonometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$\sin\theta = \frac{y}{r}$ $\qquad$ $\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}, \quad x \neq 0$ $\qquad$ $\cot\theta = \frac{x}{y}, \quad y \neq 0$

$\sec\theta = \frac{r}{x}, \quad x \neq 0$ $\qquad$ $\csc\theta = \frac{r}{y}, \quad y \neq 0$



4.4 Introduction

Because $r = \sqrt{x^2 + y^2}$ *cannot* be zero, it follows that the sine and cosine functions are defined for any real value of θ .

However, when $x = 0$, the tangent and secant of θ are undefined.

For example, the tangent of 90° is undefined. Similarly, when $y = 0$, the cotangent and cosecant of θ are undefined.

4.4 Example 1 – *Evaluating Trigonometric Functions*

Let (–3, 4) be a point on the terminal side of θ (see Figure 4.34).

Find the sine, cosine, and tangent of θ.

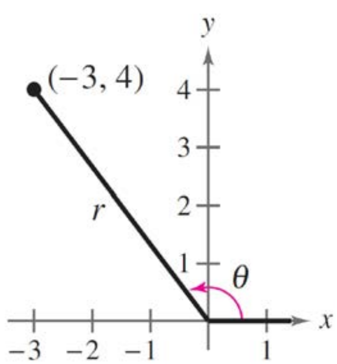


Figure 4.34

4.4 Example 1 – *Solution*

Referring to Figure 4.34, you can see that $x = –3$, $y = 4$, and

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-3)^2 + 4^2} \\ &= \sqrt{25} \\ &= 5. \end{aligned}$$

4.4 Example 1 – *Solution* cont'd

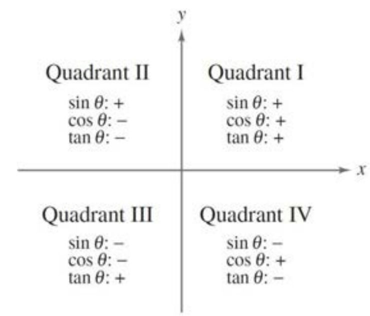
So, you have
$$\sin\theta = \frac{y}{r} = \frac{4}{5},$$
$$\cos\theta = \frac{x}{r} = -\frac{3}{5},$$
and
$$\tan\theta = \frac{y}{x} = -\frac{4}{3}.$$

4.4 Introduction

The *signs* of the trigonometric functions in the four quadrants can be determined easily from the definitions of the functions. For instance, because

$$\cos\theta = \frac{x}{r}$$

it follows that cos θ is positive wherever x > 0, which is in Quadrants I and IV.



4.4 Reference Angles

The values of the trigonometric functions of angles greater than 90° (or less than 0°) can be determined from their values at corresponding acute angles called **reference angles.**

Definition of Reference Angle

Let θ be an angle in standard position. Its **reference angle** is the acute angle θ' formed by the terminal side of θ and the horizontal axis.

4.4 Reference Angles

Figure 4.37 shows the reference angles for θ in Quadrants II, III, and IV.

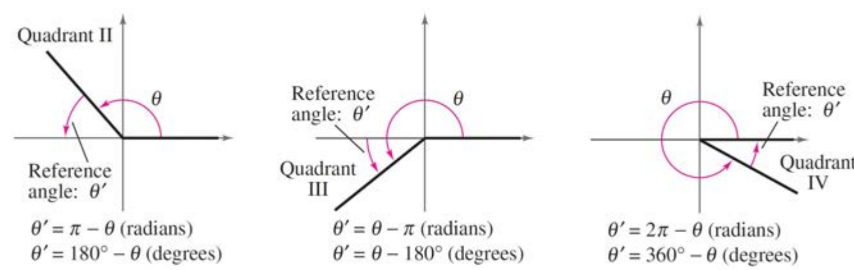


Figure 4.37

4.4 Example 4 – *Finding Reference Angles*

Find the reference angle θ'.

a. $\theta = 300°$ **b.** $\theta = 2.3$ **c.** $\theta = –135°$

Solution:

a. Because 300° lies in Quadrant IV, the angle it makes with the *x*-axis is

$$\theta' = 360° – 300° = 60°.$$

Degrees

4.4 Example 4 – *Solution*

cont'd

b. Because 2.3 lies between $\pi/2 \approx 1.5708$ and $\pi \approx 3.1416$, it follows that it is in Quadrant II and its reference angle is

$$\theta' = \pi – 2.3 \approx 0.8416.$$

Radians

c. First, determine that –135° is coterminal with 225°, which lies in Quadrant III. So, the reference angle is

$$\theta' = 225° – 180° = 45°.$$

Degrees

4.4 Example 4 – *Solution*

cont'd

Figure 4.38 shows each angle θ and its reference angle θ'.

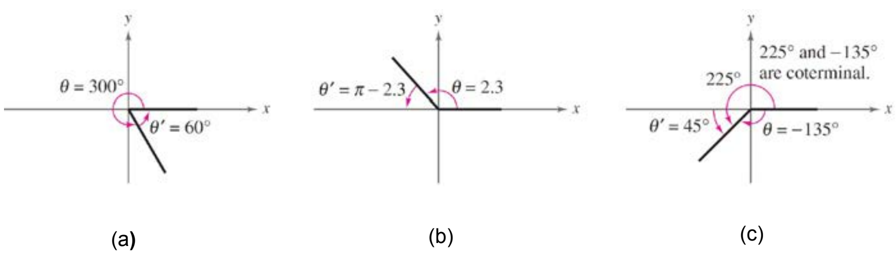


Figure 4.38

4.4 Trigonometric Functions of Real Numbers

To see how a reference angle is used to evaluate a trigonometric function, consider the point (*x*, *y*) on the terminal side of θ, as shown in Figure 4.39.

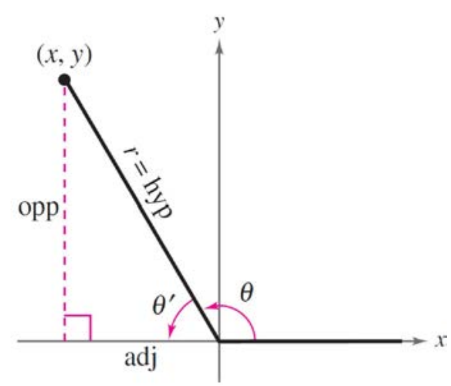


Figure 4.39

4.4 Trigonometric Functions of Real Numbers

By definition, you know that

$$\sin\theta = \frac{y}{r}$$

and

$$\tan\theta = \frac{y}{x}.$$

For the right triangle with acute angle θ' and sides of lengths $|x|$ and $|y|$, you have

$$\sin\theta' = \frac{\text{opp}}{\text{hyp}} = \frac{|y|}{r} \qquad \tan\theta' = \frac{\text{opp}}{\text{adj}} = \frac{|y|}{|x|}.$$

4.4 Trigonometric Functions of Real Numbers

So, it follows that $\sin\theta$ and $\sin\theta'$ are equal, *except possibly in sign*. The same is true for $\tan\theta$ and $\tan\theta'$ and for the other four trigonometric functions. In all cases, the sign of the function value can be determined by the quadrant in which θ lies.

Evaluating Trigonometric Functions of Any Angle

To find the value of a trigonometric function of any angle θ:

1. Determine the function value of the associated reference angle θ'.
2. Depending on the quadrant in which θ lies, affix the appropriate sign to the function value.

4.4 Trigonometric Functions of Real Numbers

For convenience, the following table shows the exact values of the trigonometric functions of special angles and quadrant angles. **(You should memorize these!)**

θ (degrees)	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$
θ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undef.	0	Undef.

Trigonometric Values of Common Angles

4.4 Example 5 – *Trigonometric Functions of Non-acute Angles*

Evaluate each trigonometric function.

a. $\cos\frac{4\pi}{3}$ **b.** tan = (–210°) **c.** $\csc\frac{11\pi}{4}$

Solution:

a. Because $\theta = 4\pi/3$ lies in Quadrant III, the reference angle is $\theta' = (4\pi/3) - \pi = \pi/3$, as shown in Figure 4.40.

Moreover, the cosine is negative in Quadrant III, so

$$\cos\frac{4\pi}{3} = (-)\cos\frac{\pi}{3} = -\frac{1}{2}.$$

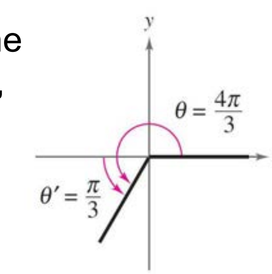


Figure 4.40

4.4 Example 5 – *Solution*

cont'd

b. Because –210° + 360° = 150°, it follows that –210° is coterminal with the second-quadrant angle 150°. Therefore, the reference angle is $\theta' = 180° - 150° = 30°$, as shown in Figure 4.41.

Finally, because the tangent is negative in Quadrant II, you have.

$$\tan(-210°) = (-)\tan 30° = -\frac{\sqrt{3}}{3}.$$

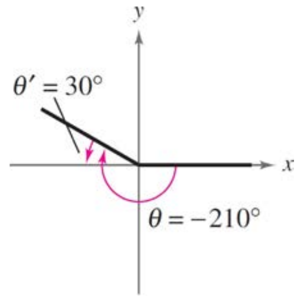


Figure 4.41

4.4 Example 5 – *Solution*

cont'd

c. Because $(11\pi/4) - 2\pi = 3\pi/4$, it follows that $11\pi/4$ is coterminal with the second-quadrant angle $3\pi/4$. Therefore, the reference angle is $\theta' = \pi - (3\pi/4) = \pi/4$, as shown in Figure 4.42.

Because the cosecant is positive in Quadrant II, you have

$$\csc\frac{11\pi}{4} = (+)\csc\frac{\pi}{4} = \frac{1}{\sin(\pi/4)} = \sqrt{2}.$$

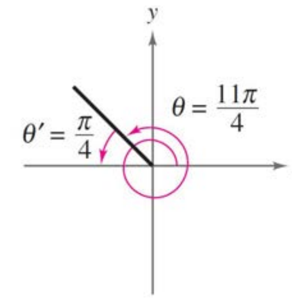


Figure 4.42

Reference Angle Worksheet 1

Determine the quadrant in which each angle lies.

1. $\frac{7\pi}{4}$	2. $\frac{11\pi}{4}$	3. $-\frac{5\pi}{6}$	4. $-\frac{13\pi}{3}$
5. –1	6. –2	7. 3	8. 2.25
9. 150°	10. 282°	11. 87.9°	12. –245.25°

Sketch each angle in standard position and state the reference angle (in the same measure as the given angle).

13. $\frac{3\pi}{4}$ Ref. ∠ = ______	14. $\frac{4\pi}{3}$ Ref. ∠ = ______	15. $-\frac{7\pi}{4}$ Ref. ∠ = ______	16. $\frac{-5\pi}{2}$ Ref. ∠ = ______

Reference Angle Worksheet 1

17. $\frac{11\pi}{6}$ Ref. ∠ = ______	18. $\frac{2\pi}{3}$ Ref. ∠ = ______	19. 4 Ref. ∠ = ______	20. –3 Ref. ∠ = ______
21. 150° Ref. ∠ = ______	22. –270° Ref. ∠ = ______	23. 405° Ref. ∠ = ______	24. –450° Ref. ∠ = ______

Reference Angle Worksheet 2

Determine two coterminal angles (one positive and one negative) for each angle. Answers can vary. Answers need to be in the same measure as the given angle.

1. $\frac{\pi}{6}$	2. $\frac{2\pi}{3}$	3. $-\frac{9\pi}{4}$
4. $-\frac{2\pi}{15}$	5. $52°$	6. $-36°$
7. $300°$	8. $-390°$	9. $114°$

Reference Angle Worksheet 2

Rewrite each angle in degree measure.

10. $\frac{3\pi}{2}$	11. $-\frac{7\pi}{6}$	12. -4π	13. $-\frac{13\pi}{60}$

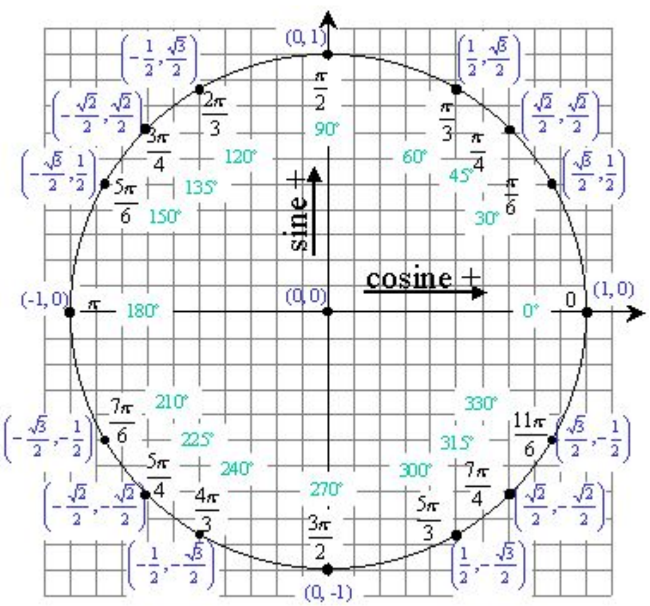
Rewrite each angle in radian measure in the following ways:

a) in terms of π

b) the rounded decimal equivalent (round three decimal places)

14. $150°$ a) b)	15. $-270°$ a) b)	16. $-240°$ a) b)	17. $20°$ a) b)

Unit Circle Notes



Trigonometric Functions Table

Degree θ	Radian θ	Sin θ	Csc θ	Cos θ	Sec θ	Tan θ	Cot θ
0	0						
30	$\frac{\pi}{6}$						
45	$\frac{\pi}{4}$						
60	$\frac{\pi}{3}$						
90	$\frac{\pi}{2}$						
120	$\frac{2\pi}{3}$						
135	$\frac{3\pi}{4}$						
150	$\frac{5\pi}{6}$						
180	π						
210	$\frac{7\pi}{6}$						
225	$\frac{5\pi}{4}$						
240	$\frac{4\pi}{3}$						
270	$\frac{3\pi}{2}$						
300	$\frac{5\pi}{3}$						
315	$\frac{7\pi}{4}$						
330	$\frac{11\pi}{6}$						
360	2π						

Unit Circle Worksheet

For #1-2, find all 6 trig functions for:

1.) $\frac{7\pi}{4}$

$\sin\frac{7\pi}{4} =$	$\csc\frac{7\pi}{4} =$
$\cos\frac{7\pi}{4} =$	$\sec\frac{7\pi}{4} =$
$\tan\frac{7\pi}{4} =$	$\cot\frac{7\pi}{4} =$

2.) $\frac{4\pi}{3}$

$\sin\frac{4\pi}{3} =$	$\csc\frac{4\pi}{3} =$
$\cos\frac{4\pi}{3} =$	$\sec\frac{4\pi}{3} =$
$\tan\frac{4\pi}{3} =$	$\cot\frac{4\pi}{3} =$

3.) Find θ if Cos $\theta = \frac{1}{2}$ and θ lies in Quadrant IV.

4.) Re-write the following in degrees:

a.) $\frac{11\pi}{6}$

b.) $\frac{2\pi}{3}$

5.) Evaluate: $\sin\frac{-17\pi}{6}$

6.) What is the reference angle for 240°? Put your answer in degrees and in radians.

7.) Evaluate sin, cos, and tan -150° without a calculator.

8.) Evaluate $\csc\frac{-3\pi}{2}$

9.) Evaluate $\cot\frac{-\pi}{2}$

10.) a.) List two angles coterminal to $-120°$.

b.) Convert all three degree measures above to radians.

c.) Determine the sine, cosine, secant, cosecant, tangent, and cotangent of all three angles above.

EXTRA CREDIT: Find the values of the 6 trig functions of θ with the given constraint:

If $\sin\theta = \frac{3}{8}$ and $\cot\theta$ is negative. *HINT: Use a triangle instead of a unit circle.*

Defining Kinematics

Kinematics is the study of the motion of points, objects, and groups of objects without considering the causes of its motion.

LEARNING OBJECTIVES

Define kinematics

KEY TAKEAWAYS

Key Points

- To describe motion, kinematics studies the trajectories of points, lines and other geometric objects.
- The study of kinematics can be abstracted into purely mathematical expressions.
- Kinematic equations can be used to calculate various aspects of motion such as velocity, acceleration, displacement, and time.

Key Terms

- **kinematics**: The branch of mechanics concerned with objects in motion, but not with the forces involved.

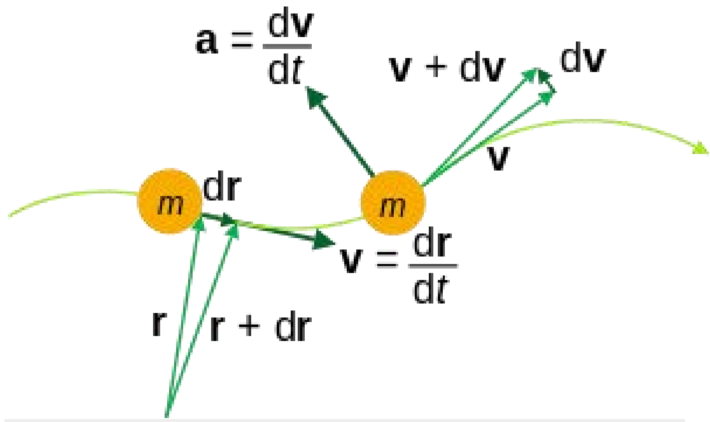
Kinematics is the branch of classical mechanics that describes the motion of points, objects and systems of groups of objects, without reference to the causes of motion (i.e., forces). The study of kinematics is often referred to as the “geometry of motion.”

Objects are in motion all around us. Everything from a tennis match to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. Even in inanimate objects there is continuous motion in the vibrations of atoms and molecules. Interesting questions about motion can arise: how long will it take for a space probe to travel to Mars? Where will a football land if thrown at a certain angle? An understanding of motion, however, is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

To describe motion, kinematics studies the trajectories of points, lines and other geometric objects, as well as their differential properties (such as velocity and acceleration). Kinematics is used in astrophysics to describe the motion of celestial bodies and systems; and in mechanical engineering, robotics and biomechanics to describe the motion of systems composed of joined parts (such as an engine, a robotic arm, or the skeleton of the human body).

A formal study of physics begins with kinematics. The word “kinematics” comes from a Greek word “kinesis” meaning motion, and is related to other English words such as “cinema” (movies) and “kinesiology” (the study of human motion).

Kinematic analysis is the process of measuring the kinematic quantities used to describe motion. The study of kinematics can be abstracted into purely mathematical expressions, which can be used to calculate various aspects of motion such as velocity, acceleration, displacement, time, and trajectory.



Kinematics of a particle trajectory: Kinematic equations can be used to calculate the trajectory of particles or objects. The physical quantities relevant to the motion of a particle include: mass m, position r, velocity v, acceleration a.

Reference Frames and Displacement

In order to describe an object's motion, you need to specify its position relative to a convenient reference frame.

LEARNING OBJECTIVES

Evaluate displacement within a frame of reference.

KEY TAKEAWAYS

Key Points

- Choosing a frame of reference requires deciding where the object's initial position is and which direction will be considered positive.
- Valid frames of reference can differ from each other by moving relative to one another.
- Frames of reference are particularly important when describing an object's displacement.
- Displacement is the change in position of an object relative to its reference frame.

Key Terms

- **displacement**: A vector quantity that denotes distance with a directional component.
- **frame of reference**: A coordinate system or set of axes within which to measure the position, orientation, and other properties of objects in it.

In order to describe the motion of an object, you must first describe its position — where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of objects related to its position to or from Earth. Mathematically, the position of an object is generally represented by the variable *x*.

Frames of Reference

There are two choices you have to make in order to define a position variable *x*. You have to decide where to put $x = 0$ and which direction will be positive. This is referred to as choosing a coordinate system, or choosing a frame of reference. As long as you are consistent, any frame is equally valid. But you don't want to change coordinate systems in the middle of a calculation. Imagine sitting in a train in a station when suddenly you notice that the station is moving backward. Most people would say that they just failed to notice that the train was moving — it only *seemed* like the station was moving. But this shows that there is a *third* arbitrary choice that goes into choosing a coordinate system: valid frames of reference can differ from each other by moving relative to one another. It might seem strange to use a coordinate system moving relative to the earth — but, for instance, the frame of reference moving along with a train might be far more convenient for describing things happening inside the train. Frames of reference are particularly important when describing an object's displacement.

OPTIONAL YouTube Video: FRAMES OF REFERENCE by Professor Hume and Professor Donald Ivey of the University of Toronto

In this classic film, Professors Hume and Ivey cleverly illustrate reference frames and distinguish between fixed and moving frames of reference.

Frames of Reference (1960) Educational Film: Frames of Reference is a 1960 educational film by Physical Sciences Study Committee. The film was made to be shown in high school physics courses. In the film University of Toronto physics professors Patterson Hume and Donald Ivey explain the distinction between inertial and nonintertial frames of reference, while demonstrating these concepts through humorous camera tricks. For example, the film opens with Dr. Hume, who appears to be upside down, accusing Dr. Ivey of being upside down. Only when the pair flip a coin does it become obvious that Dr. Ivey — and the camera — are indeed inverted.

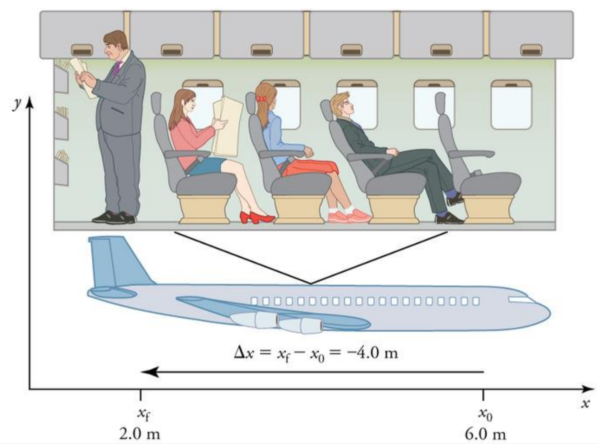
Displacement

Displacement is the change in position of an object relative to its reference frame. For example, if a car moves from a house to a grocery store, its displacement is the relative distance of the grocery store to the reference frame, or the house. The word "displacement" implies that an object has moved or has been displaced. Displacement is the change in position of an object and can be represented mathematically as follows:

$\Delta x=xf-x0\Delta x=xf-x0$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

shows the importance of using a frame of reference when describing the displacement of a passenger on an airplane.



Displacement in Terms of Frame of Reference: A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far).

Acceleration

An often confused quantity, acceleration has a meaning much different than the meaning associated with it by sports announcers and other individuals. The definition of acceleration is:

- **Acceleration** is a vector quantity that is defined as the rate at which an object changes its velocity. An object is accelerating if it is changing its velocity.

Sports announcers will occasionally say that a person is accelerating if he/she is moving fast. Yet acceleration has nothing to do with going fast. A person can be moving very fast and still not be accelerating. Acceleration has to do with changing how fast an object is moving. If an object is not changing its velocity, then the object is not accelerating. The data at the right are representative of a northward-moving accelerating object. The velocity is changing over the course of time. In fact, the velocity is changing by a constant amount - 10 m/s - in each second of time. Anytime an object's velocity is changing, the object is said to be accelerating; it has an acceleration.

Time	Velocity
0 s	0 m/s, No
1 s	10 m/s, No
2 s	20 m/s, No
3 s	30 m/s, No
4 s	40 m/s, No
5 s	50 m/s, No

The Meaning of Constant Acceleration

Sometimes an accelerating object will change its velocity by the same amount each second. As mentioned in the previous paragraph, the data table above show an object changing its velocity by 10 m/s in each consecutive second. This is referred to as a **constant acceleration** since the velocity is changing by a constant amount each second. An object with a constant acceleration should not be confused with an object with a constant velocity. Don't be fooled! If an object is changing its velocity -whether by a constant amount or a varying amount - then it is an accelerating object. And an object with a constant velocity is not accelerating. The data tables below depict motions of objects with a constant acceleration and a changing acceleration. Note that each object has a changing velocity.

Accelerating Objects are Changing Their Velocity ...

... by a constant amount each second ...

Time (s)	Velocity (m/s)
0	0
1	4
2	8
3	12
4	16

...in which case, it is referred to as a constant acceleration.

... or by a changing amount each second ...

Time (s)	Velocity (m/s)
0	0
1	1
2	4
3	5
4	7

...in which case, it is referred to as a non-constant acceleration.

Since accelerating objects are constantly changing their velocity, one can say that the distance traveled/time is not a constant value. A falling object for instance usually accelerates as it falls. If we were to observe the motion of a **free-falling object** (free fall motion will be discussed in detail later), we would observe that the object averages a velocity of approximately 5 m/s in the first second, approximately 15 m/s in the second second, approximately 25 m/s in the third second, approximately 35 m/s in the fourth second, etc. Our free-falling object would be constantly accelerating. Given these average velocity values during each consecutive 1-second time interval, we could say that the object would fall 5 meters in the first second, 15 meters in the second second (for a total distance of 20 meters), 25 meters in the third second (for a total distance of 45 meters), 35 meters in the fourth second (for a total distance of 80 meters after four seconds). These numbers are summarized in the table below.

Time Interval	Velocity Change During Interval	Ave. Velocity During Interval	Distance Traveled During Interval	Total Distance Traveled from 0 s to End of Interval
0 – 1.0 s	0 to ~10 m/s	~5 m/s	~5 m	~5 m
1.0 – 2.0 s	~10 to 20 m/s	~15 m/s	~15 m	~20 m
2.0 – 3.0 s	~20 to 30 m/s	~25 m/s	~25 m	~45 m
3.0 – 4.0 s	~30 to 40 m/s	~35 m/s	~35 m	~80 m

Note: The ~ symbol as used here means approximately.

This discussion illustrates that a free-falling object that is accelerating at a constant rate will cover different distances in each consecutive second. Further analysis of the first and last columns of the data above reveal that there is a square relationship between the total distance traveled and the time of travel for an object starting from rest and moving with a constant acceleration. The total distance traveled is directly proportional to the square of the time. As such, if an object travels for twice the time, it will cover four times (2^2) the distance; the total distance traveled after two seconds is four times the total distance traveled after one second. If an object travels for three times the time, then it will cover nine times (3^2) the distance; the distance traveled after three seconds is nine times the distance traveled after one second. Finally, if an object travels for four times the time, then it will cover 16 times (4^2) the distance; the distance traveled after four seconds is 16 times the distance traveled after one second. For objects with a constant acceleration, the distance of travel is directly proportional to the square of the time of travel.

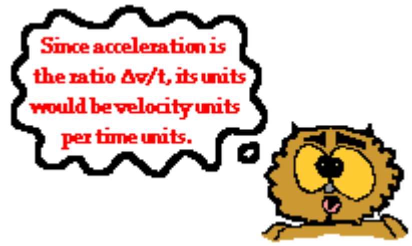
Calculating the Average Acceleration

The average acceleration (**a**) of any object over a given interval of time (**t**) can be calculated using the equation

$$\text{Ave. acceleration} = \frac{\Delta \text{velocity}}{\text{time}} = \frac{v_f - v_i}{t}$$

This equation can be used to calculate the acceleration of the object whose motion is depicted by the velocity-time data table above. The velocity-time data in the table shows that the object has an acceleration of 10 m/s/s. The calculation is shown below.

$$a = \frac{v_f - v_i}{t} = \frac{50\text{ m/s} - 0\text{ m/s}}{5\text{ s}} = \frac{10\text{ m/s}}{1\text{ s}}$$



Acceleration values are expressed in units of velocity/time. Typical acceleration units include the following:

m/s/s
mi/hr/s
km/hr/s
m/s^2

These units may seem a little awkward to a beginning physics student. Yet they are very reasonable units when you begin to consider the definition and equation for acceleration. The reason for the units becomes obvious upon examination of the acceleration equation.

$$a = \frac{\Delta \text{velocity}}{\text{time}}$$

Since acceleration is a velocity change over a time, the units on acceleration are velocity units divided by time units - thus (m/s)/s or (mi/hr)/s. The (m/s)/s unit can be mathematically simplified to m/s^2.

The Direction of the Acceleration Vector

Since acceleration is a vector quantity, it has a direction associated with it. The direction of the acceleration vector depends on two things:

- whether the object is speeding up or slowing down
- whether the object is moving in the + or - direction

The general principle for determining the acceleation is:

If an object is slowing down, then its acceleration is in the opposite direction of its motion.

This general principle can be applied to determine whether the sign of the acceleration of an object is positive or negative, right or left, up or down, etc. Consider the two data tables below. In each case, the acceleration of the object is in the *positive* direction. In Example A, the object is moving in the *positive* direction (i.e., has a *positive* velocity) and is speeding up. When an object is speeding up, the acceleration is in the same direction as the velocity. Thus, this object has a **positive acceleration**. In Example B, the object is moving in the *negative* direction (i.e., has a negative velocity) and is slowing down. According to our general principle, when an object is slowing down, the acceleration is in the opposite direction as the velocity. Thus, this object also has a **positive acceleration**.

Example A

Time (s)	Velocity (m/s)
0	0
1	2
2	4
3	6
4	8

Example B

Time (s)	Velocity (m/s)
0	-8
1	-6
2	-4
3	-2
4	0

These are both examples of positive acceleration.

This same general principle can be applied to the motion of the objects represented in the two data tables below. In each case, the acceleration of the object is in the *negative* direction. In Example C, the object is moving in the *positive* direction (i.e., has a *positive* velocity) and is slowing down. According to our principle, when an object is slowing down, the acceleration is in the opposite direction as the velocity. Thus, this object has a **negative acceleration**. In Example D, the object is moving in the *negative* direction (i.e., has a *negative* velocity) and is speeding up. When an object is speeding up, the acceleration is in the same direction as the velocity. Thus, this object also has a **negative acceleration**.

Example C

Time (s)	Velocity (m/s)
0	8
1	6
2	4
3	2
4	0

Example D

Time (s)	Velocity (m/s)
0	0
1	-2
2	-4
3	-6
4	-8

These are both examples of negative acceleration.

Observe the use of positive and negative as used in the discussion above (Examples A - D). In physics, the use of positive and negative always has a physical meaning. It is more than a mere mathematical symbol. As used here to describe the velocity and the acceleration of a moving object, positive and negative describe a direction. Both velocity and acceleration are vector quantities and a full description of the quantity demands the use of a directional adjective. North, south, east, west, right, left, up and down are all directional adjectives. Physics often borrows from mathematics and uses the + and - symbols as directional adjectives. Consistent with the mathematical convention used on number lines and graphs, positive often means to the right or up and negative often means to the left or down. So to say that an object has a negative acceleration as in Examples C and D is to simply say that its acceleration is to the left or down (or in whatever direction has been defined as negative). Negative accelerations do not refer acceleration values that are less than 0. An acceleration of -2 m/s/s is an acceleration with a magnitude of 2 m/s/s that is directed in the negative direction.

Check Your Understanding

To test your understanding of the concept of acceleration, consider the following problems and the corresponding solutions. Use the equation for acceleration to determine the acceleration for the following two motions.

Practice A

Time (s)	Velocity (m/s)
0	0
1	2
2	4
3	6
4	8

Practice B

Time (s)	Velocity (m/s)
0	8
1	6
2	4
3	2
4	0

Benchmark Standard	History 2a: Students will develop and implement effective research strategies for investigating a given historical topic.
Grade Band	11-12
Vocabualry / Key Concepts:	See the vocabulary listed in the documents.

~This is a SHEG lesson modified by CSD for Home~

Central Historical Question: Why did the U.S. invade Cuba?

ACTIVITY 1: Read the transcript and answer the question that follows on a separate sheet of paper:

The following is the transcript from the video: http://historicalthinkingmatters.org/spanishamericanwar/
The island of Cuba lies only 90 miles from the United States. A colony of Spain since 1492, Cuba exported sugar, fruit and tobacco to the United States and the rest of the world. American investors by 1880, had spent millions of dollars in Cuba. In Cuba itself, movements for independence from Spain had a very long history. By 1880, the Cuban insurrection had become a crisis for the Spanish government. On January 28, 1898, the USS Maine had entered Havana Harbor, Cuba. The Maine was one of four state of the art battle ships in the growing US fleet. The ship was a great source of pride and a symbol of America's increasing world influence. President William McKinley ordered the Maine in to Havana, but just why he sent it is unclear. Some argue McKinley saw the order as part of normal diplomatic exchange with Spain. Others argue that McKinley wanted to encourage the Cuban rebels and demoralize the Spanish. McKinley may also have wanted to protect American property from both the Spanish and the rebels. BOOM! On February 15, 1898, the Maine exploded – a devastating blast that ripped the ship's hull apart. The Maine quickly sank. 266 Americans died in the explosion and fire or drowned. Across the country Americans asked in public and private "Who sank the Maine?" 3 months later, the United States declared war on Spain and began preparing troops for an invasion of Cuba. Your task is to figure out why… Why did the United States Invade Cuba?

1. In June 1898, the U.S. sent troops into Cuba. Over the next few days, you are going to investigate why. Based on the transcript, list all the different possible reasons why the U.S. chose to invade Cuba. These "guesses" are your hypotheses. You will answer questions based on your list of hypotheses at the end of the lesson.

ACTIVITY 2: Read the Song "Awake United States" and fill in the section on the Graphic Organizer (page 4) that corresponds with the song.

"Awake United States"

This song was rushed into print between the sinking of the Maine on February 15, 1898, and the declaration of war on April 25, 1898.

Eagle soar on high, and sound the battle cry!
And how proudly sailed the warship Maine,
a Nation's pride, without a stain!
A wreck she lies, her sailors slain.
By two-faced butchers, paid by Spain!
Eagle soar on high,
And sound the battle cry
Wave the starry flag!
In mud it shall not drag!

ACTIVITY 3: Read Document A and on answer the Guiding Questions (page 3) that correspond with Document A.

Document A: Reconcentration Camps

By the late 1800s, the Spanish were losing control of Cuba. Concerned about insurrection in the countryside, they moved rural Cubans to "reconcentration" camps where the Spanish claimed they would be better able to protect them. U.S. ***Consul-General*** *Fitzhugh Lee forwarded the following account of the conditions of the camps to the U.S. Assistant Secretary of State on November 27, 1897. Lee said the author of the note was "a man of integrity and character."*

[W]e will relate to you what we saw with our own eyes:

Four hundred and sixty women and children thrown on the ground, heaped **pell-mell** as animals, some in a dying condition, others sick and others dead. . . .

There is still alive the only living witness, a young girl of 18 years, whom we found seemingly lifeless on the ground; on her right-hand side was the body of a young mother, cold and rigid, but with her young child still alive clinging to her dead breast; on her left-hand side was also the corpse of a dead woman holding her son in a dead embrace. . . .

The circumstances are the following: complete **accumulation** of bodies dead and alive, so that it was impossible to take one step without walking over them; the greatest **want** of cleanliness, want of light, air, and water; the food lacking in quality and quantity what was necessary to sustain life. . . . From all this we deduct that the number of deaths among the **reconcentrados** has amounted to 77 per cent.

Source: *Unsigned note that was included in a telegram sent by Fitzhugh Lee, U.S. Consul-General in Cuba, to the U.S. Assistant Secretary of State November 27, 1897.*

Vocabulary:

Consul-general: a government official living in a foreign country charged with overseeing the protection of U.S. citizens and promoting trade

Pell-mell: state of disorder

Accumulation: pile

Want: lack

Reconcentrados: the reconcentration camp prisoners

Citations Document A

Lee, Fitzhugh. Fitzhugh Lee, US Consulate-General in Cuba, to Assistant Secretary of State Day, 27 November 1897. In Message from the President of the United States, transmitting, in response to the resolution of the House of Representatives, Dated February 14, 1898, Calling for information in respect to the condition of the reconcentrados in Cuba, the state of the war and the country, and the prospects of projected autonomy in that island. Washington, DC: Government Printing Office, 1898. 9-11. Retrieved from http://historicalthinkingmatters.org/spanishamericanwar/0/inquiry/main/resources/2/

ACTIVITY 4: Read Document B and answer the Guiding Questions (page 3) that correspond with Document B.

Document B: March of the Flag

The following is an excerpt from Albert J. Beveridge's speech, delivered September 16, 1898. Beveridge gave this speech while he was campaigning to become a senator for Indiana. The speech helped him win the election and made him one of the leading advocates of American expansion.

Fellow citizens, it is a noble land that God has given us; a land that can feed and clothe the world;. . . . It is a mighty people that he has planted on this soil . . . It is a glorious history our God has bestowed upon his chosen people; . . .a history of soldiers who carried the flag across the blazing deserts and through the ranks of hostile mountains, even to the gates of sunset. . . .

The Opposition tells us that we ought not to govern a people without their consent. I answer: The rule of liberty that all just government derives its authority from the consent of the governed, applies only to those who are capable of self-government. I answer, We govern the Indians without their consent, we govern our territories without their consent, we govern our children without their consent.

They ask us how we will govern these new possessions. I answer: If England can govern foreign lands, so can America. If Germany can govern foreign lands, so can America. . . .

What does all this mean for every one of us? It means opportunity for all the glorious young manhood of the republic, the most **virile**, ambitious, impatient, **militant** manhood the world has ever seen. It means that the resources and the commerce of these immensely rich **dominions** will be increased. . . .

In Cuba, alone, there are 15,000,000 acres of forest unacquainted with the axe. There are exhaustless mines of iron. . . . There are millions of acres yet unexplored. . . . It means new employment and better wages for every laboring man in the Union. . . .

Ah! as our commerce spreads, the flag of liberty will circle the globe. . . . **Benighted** peoples will know that the voice of Liberty is speaking, at last, for them; that civilization is dawning, at last, for them. . . .

Fellow Americans, we are God's chosen people. . . . Source: Albert J. Beveridge's Senate campaign speech, September 16, 1898.

Vocabulary:

Virile: having strength and energy

Militant: aggressive

Dominions: controlled territories

Benighted: pitifully ignorant

Citations Document B

Beveridge, Albert J. "March of the Flag." (September 16, 1898, Indiana). In The Meaning of the Times, and Other Speeches. Indianapolis: Bobbs-Merrill, 1908. 47-57. Retrieved from http://historicalthinkingmatters.org/spanishamericanwar/0/inquiry/main/resources/7/

Guiding Questions:

Document A: Reconcentration Camps

1. **Sourcing**: Given that in the U.S. there was an ongoing debate about whether the U.S. should intervene in Cuba, why might Lee have chosen to send this account to Washington?
2. **Close Reading**: Notice the graphic descriptions of the account. How do these details about the camp conditions affect you as you read? Why might these descriptions be so detailed?
3. **Contextualization**: How do you think U.S. government officials might have reacted to this description of the reconcentration camps?

Document B: March of the Flag

1. **Sourcing**: This speech is part of a political campaign. How does that influence what you can expect of it?
2. **Close Reading**: What do the following phrases suggest about Beveridge's view of Americans as compared with people of other nations?
    a) "noble land that God has given us"
    b) "applies only to those who are capable of self-government"
    c) "civilization is dawning, at last, for them"
3. **Contextualizing:** According to Beveridge, what else was going on in the U.S. and the rest of the world that made expansion a good idea?

Activity 5: If you need to, reread Document A and Document B. Complete the Graphic Organizer sections that correspond with Document A and Document B.

Graphic Organizer for “Why Did the United States Invade Cuba in 1898?”

Document Name	Date / Author	According to this document, why did the United States invade Cuba?	Provide evidence from the source that supports these reasons.
Awake United States			
Document A: Reconcentration Camps			
Document B: March Of the Flag			

ACTIVITY 6: Look back at your list of hypotheses from Activity 1, and answer the following questions:

1. How do these sources support or contest any of the hypotheses? Explain.
2. Are any hypotheses more convincing to you now? Explain.
3. Do you think the U.S. invaded Cuba for humanitarian reasons? Explain why or why not.
4. Do you think any new hypotheses should be added? Do you think any hypotheses should be changed or eliminated? Explain with evidence why you think the U.S. invaded Cuba.